Multiple Quantization and the Concept of Information[1]

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Abstract

The understanding of the meaning of quantization seems to be the main problem in understanding quantum structures. In this paper first the difference between quantized particle vs. radiation fields in the formalism of canonical quantization is discussed. Next von Weizsäcker's concept of "multiple quantization" which leads to an understanding of quantization as an iteration of probability theory is explained. Finally a connection between quantization and the idea of a "general theory of information" is considered. This brings together semantic information with the different levels of quantization and expresses the philosophical attitude of this paper concerning the interpretation of quantum theory.

1 Quantum Field Theory

When quantizing a field one has to differentiate between the quantization of a classical radiation field such as the electromagnetic field and a quantum field such as the Schrödinger, Klein-Gordon, or Dirac field. Only in the latter cases is the field quantization indeed a "second quantization". From a fundamental point of view the electromagnetic field is one of the gauge fields in physics which describes one of the fundamental forces, whereas the Dirac field describes the fundamental fermions such as quarks and leptons. For the sake of simplicity I will only consider the electron and the photon as examples of the fundamental particle and gauge fields. Physically there is a clear difference between them: the electron is a fermionic field of matter which describes particles and the photon is a bosonic gauge field which describes interaction.

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On the other hand the canonical formalism for the field quantization seems to make no difference between this physical meaning of the fields: they are both quantized fields and therefore several authors maintain that there is no more wave-particle-dualism on the level of quantum field theory.

According to the usual interpretation a quantized field is understood as a totality of field quanta which can be created and annihilated. Quantum field theory therefore is essentially a many-particle theory.

1.1 The Dirac Field

The quantization of the Dirac spinors ψ, $\bar{\psi}$ leads to the operators

$$\begin{aligned} \hat{\psi}(x) &= \sum_{\pm s} \int \frac{d^3p}{\sqrt{(2\pi)^3 \frac{E}{m}}} \left(\hat{b}(p,s) u(p,s) e^{-ipx} + \hat{d}^+(p,s) v(p,s) e^{ipx} \right) \\ \hat{\bar{\psi}}(x) &= \sum_{\pm s} \int \frac{d^3p}{\sqrt{(2\pi)^3 \frac{E}{m}}} \left(\hat{b}^+(p,s) \bar{u}(p,s) e^{ipx} + \hat{d}(p,s) \bar{v}(p,s) e^{-ipx} \right) \end{aligned} \tag{1}$$

which describe the electron and the positron field, i.e, particles and antiparticles. The operators $\hat{b}^+(p,s)$, $\hat{b}(p,s)$, $\hat{d}^+(p,s)$ and $\hat{d}(p,s)$ obey the commutation relations

$$\left\{ \hat{b}(p,s), \hat{b}^+(p',s') \right\} = \left\{ \hat{d}(p,s), \hat{d}^+(p',s') \right\} = \delta_{ss'} \delta^3(\vec{p} - \vec{p}') \tag{2}$$

and zero otherwise. Usually the canonical quantization procedure would lead to Bose commutation relations instead of (2), which for Dirac spinors violate microcausality. Therefore anticommutation relations are needed which lead to fermions, in agreement with experience.

Thus the Dirac field turns out to be an essentially complex-valued field, i.e., the operators (1) are non-Hermitian. They do not describe quantities which are observed. Measurable properties of the quantized Dirac field can only be expressed in bilinear terms of the fields. One therefore has to look at the operator of the probability density current

$$\hat{\jmath}^\mu = \hat{\bar{\psi}} \gamma^\mu \hat{\psi} \tag{3}$$

which is conserved

$$\partial_\mu \hat{\jmath}^\mu = 0. \tag{4}$$

1.2 The Electromagnetic Field

In the case of the electromagnetic field the observed quantities are the field forces $\vec{E}$ and $\vec{B}$ covariantly expressed by the tensor

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu. \tag{5}$$

For the quantization of the electromagnetic field, however, one should start from the potential A^ν because it appears in the interaction terms and the transition amplitudes. One then gets the operator

$$\hat{A}_\mu(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2k_0}} (\hat{a}_\mu(\vec{k})e^{-ikx} + \hat{a}^+_\mu(\vec{k})e^{ikx}) \tag{6}$$

where the Fourier amplitudes $\hat{a}_\mu(\vec{k})$, $\hat{a}^+_\mu(\vec{k})$ are to be regarded as photon annihilation and creation operators. The canonical formalism leads to the commutation relations

$$\left[\hat{a}(\vec{k}), \hat{a}^+(\vec{k}')\right] = \delta(\vec{k} - \vec{k}') \tag{7}$$

and zero otherwise. In this paper I do not want to go into details concerning the special problems of quantizing the electromagnetic field in a covariant manner and to hold also only the two physical transversal polarization states of the photon instead of the four degrees of freedom of the covariant potential A_μ. These problems are related to the gauge freedom of A_μ and lead to the Gupta-Bleuler quantization.

Our interest is related to the question of whether the quantized electromagnetic field can be regarded as a totality of photons in the same manner as the Dirac field can be for electrons. In this context two differences appear. First, in contrast to the quantized Dirac field (1), the operator (6) is Hermitian. This is an expression of the measurability of the quantized field forces $\hat{\vec{E}}$ and $\hat{\vec{B}}$ - of course, only within the scope of the uncertainty relations, which are compatible with (7). This is discussed in a famous paper by Bohr and Rosenfeld (1950).

Second, the relation analogous to (4) does not hold for the free photon field because the operator

$$\hat{\jmath}^\nu = \partial_\mu \hat{F}^{\mu\nu} = \Box \hat{A}^\nu - \partial_\mu(\partial^\nu \hat{A}^\mu) = 0 \tag{8}$$

vanishes. This consequently means that there is no conservation law for the number of photons. In other words the total number of photons is uncertain. One therefore has to draw the conclusion that the concept of a well-defined particle density (expressed by the number operator) is not meaningful in the same way for the photon field as it is for the electron field. Or, in the words of Pauli (1933, p. 579): *"... daß für das Photonfeld ... der Begriff der raum-zeitlich-lokalen Teilchendichte $W(\vec{x}, t)$ nicht sinnvoll existiert"* [*... that for the photon field ... the notion of a particle density $W(\vec{x}, t)$ located in space-time has no meaningful existence* (translation by the author)].

2 Multiple Quantization

In the 1950s von Weizsäcker (1955; 1958; von Weizsäcker, von Weizsäcker et al., 1958) introduced both the idea of what was later called the quantum theory of ur-alternatives

("ur-theory") and his concept of multiple quantization. Both ideas are related to each other.

2.1 The Quantum Theory of Ur-Alternatives

The ur-theory is a program to understand the unity of physics and is based on the simplest possible object which can be found in quantum theory: the quantized binary alternative (shortly "ur-object" or "ur"). It is not the intention of this paper to describe the structure of ur-theory (von Weizsäcker, 1985, Chapters 9 and 10); only a short introduction to the basic idea shall be given. In ur-theory the assumption is that the three-dimensionality of position space is a consequence of the symmetry group of the ur, which is essentially $SU(2)$. Moreover, the homogeneous space of $SU(2)$, which is $\mathbb{S}^3$, can be looked upon as a model of our cosmos. The argument for this is that if quantum theory gives the fundamental structure of any physical theory, then any physical object must be described by a Hilbert space which in any case can be embedded into a tensor product space of urs. Thus any physical object can be trivially build up from urs and therefore the symmetry properties of the position space have to be the symmetry properties of urs. In ur-theory the line of argument is turned around: the symmetry of position space is regarded as a consequence of the symmetry of urs. This points toward a close connection between empirical alternatives and their testability in space (Lyre, 1995).

Thus an ur-alternative turns out to be the fundamental object in physics. But in standard physics we deal with particles and fields as described above. This leads to the concept of multiple quantization.

2.2 The Statistical Interpretation of Quantization

Let us now ask about the meaning of quantization and suppose quantum theory to be fundamental. Thus we do not want to introduce quantum theory from classical mechanics via the "correspondence principle". Instead we will follow von Weizsäcker's proposal of the connection between quantum theory and probability theory.

Let us call the n possible answers, excluding each other,

$$a_k \quad (k = 1...n) \tag{9}$$

to a given question an *n-fold-alternative*. Then the complex numbers

$$\psi_k \quad (k = 1...n) \tag{10}$$

should be the corresponding truth values. If ψ is normalized, then

$$p_k = \psi_k^* \psi_k \tag{11}$$

is the probability to find a_k. Now probability can be defined as the *expectation value of a relative frequency* $f_k = \frac{n_k}{n}$ (Drieschner, 1979)

$$p_k = E(f_k) = \sum_{f_k} p(f_k)\ f_k. \tag{12}$$

This definition fits the fact that in real measurements only the number n_k of the occurrences of a_k in a series of n experiments is observed. Therefore in quantum theory n_k has to be regarded as an operator. It turns out that

$$\hat{n}_k = \hat{\psi}_k^+ \hat{\psi}_k \tag{13}$$

is a suitable choise, whereas the new opeators $\hat{\psi}_k^+$, $\hat{\psi}_k$ obey certain commutation relations and act as creation and annihilation operators of states ψ_k.

From

$$\left\langle \hat{\psi} \middle| \hat{\psi} \right\rangle = \sum_k \hat{\psi}_k^+ \hat{\psi}_k = \sum_k \hat{n}_k = \hat{n} \tag{14}$$

it follows that *the operator $\hat{\psi}$ of the next level of quantization must be interpreted as a totality of n objects of the level below - each one described by a single wave function ψ.* One therefore is led to a statistical interpretation of the quantization procedure.

The definition (12) has yet another consequence. On the first view it looks like a circular definition: the probability p_k is defined by another probability $p(f_k)$. But one has to keep in mind that $p(f_k)$ is a probability of the next-higher level. It describes the probability to find a series of experiments (where a_k was found with the relative frequency f_k) in a series of series of experiments. This new probability again refers to a probability of a higher level and so on. Thus the step-like structure of probability theory appears and, because of the connection between quantization and probability as described above, this leads - by the same argument - to a step-like structure of quantization. Therefore there should be not only two, but multiple levels of quantization (von Weizsäcker, 1973).

2.3 Multiple Quantization in Ur-Theory

In ur-theory one starts with a simple, empirically testable, binary alternative a_r ($r = 1, 2$). The first quantization of a_r leads to the complex spinor u_r. On the second level of quantization one has to introduce the ur-operators $\hat{u}_r^+$, $\hat{u}_r$. It was found that the momentum states of massless and massive particles can be build up from creation and annihilation operators of urs and anti-urs ($r = 1, ...4$). The appropriate commutation relations for these operators represent a parabose-statistics of urs (Görnitz et al., 1992). Thus the quantum field theory of particles such as quarks and leptons (see Section 1.1) appears - in the light of ur-theory - as the third level of quantization of the alternative a_r. In view of the problems with the statistical interpretation of the quantized electromagnetic

field (see Section 1.2), the question arises of whether the photon should be build up from urs in the same way as particles, or, if not, in what other way? Surely the gauge theoretic character of the interaction fields must be explained in ur-theory, but this leads to open questions concerning the problem of interaction in ur-theory which will not be discussed in this paper.

3 A General Theory of Information

From the interpretational point of view the theory of ur-objects must be regarded as a *quantum theory of information* consequently thought to its end. An ur-alternative represents exactly *one bit of potential information*. The question now is: what is the meaning of the different levels of quantization within the framework of a quantum theory of information?

For this purpose one has to be aware of the difference between *syntactic* and *semantic information.* I call syntactic information an *amount of structural distinguishability* which can be measured in *bits*. Beyond this the semantic aspect of information takes care of the fact that information only exists under a certain concept or on a certain semantic level. For example, a letter printed on a paper refers to different amounts of information if it is regarded under the concept "letter of an alphabet of a certain language" or under the concept "molecules of printer's ink". The statistical interpretation of quantization stresses the importance of the forming of collectives, i.e., a wave function of a certain level of quantization describes a totality of objects of the level below. This is in a certain way analogous to the forming of concepts, e.g., the concept "animal" describes the totality of cats, dogs, snakes, elephants, mosquitoes and so on.

In the light of multiple quantization in ur-theory we get the following semantic levels: An ur-object represents the simplest structural distinction which can be made in empirical science: a spatial yes-no alternative - one bit of information. The next level of quantization refers to particles. The concept "particle" describes a totality of urs which are to be regarded as the field quanta of a particle. At the next level , the level of quantum field theory, the objects of the level below, i.e., particles, become field quanta for themselves, i.e., the former "concepts" must now be regarded as "syntactic elements" under the new concept of the quantized particle field.

This is exactly my concluding assumption: quantum theory must be regarded as a general theory of information and quantization has to be understood as the forming of concepts or semantic levels which are necessary for the existence of information in general. In ur-theory the problem still remains of what status the interaction fields in this information-theoretic view will have.

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